Tensegrity Structures

Tensegrity system dynamics is a subset of the class of multi-body dynamics which includes cylindrical rigid bodies (bars) and elastic members (strings) arranged in a stabilizable topology (Skelton & Oliveira, 2009). Tensegrity structures appeared in earlier artworks by Karl Ioganson in 1921 and Kenneth Snelson in 1948 (Sultan, 2009). The term tensegrity was first coined by Buckminster Fuller (1959), and is a portmanteau of “tensional integrity” and Snelson (1965) filed the first patents on this topic. Tensegrity structures are created by methodically arranging tenseile members (strings) and compressive members (bars) to form a stable system. Skelton & Oliveira (2009) define a tensegrity structure as a “class-1” tensegrity system if none of the compressive members are connected, if, on the other hand, $k$ compressive members are connected at a node, this is referred to as a “class-$k$” tensegrity system. The multiple compressive members are connected through ball joints, causing all the members in a tensegrity structure to be axially loaded, i.e., no moment is present on any individual member. Tensegrity structures can also be prestressed to have uni-directional loading for all the individual members giving the freedom to design tension and compression members separately. This freedom provides good structural efficiency (high strength-to-mass ratio) to tensegrity structures. A particular topology of tensegrity structure can provide various self-equilibrium solutions corresponding to different values of prestress in the structure. The various stable prestress values provide a domain set to minimize the mass of the structure. This minimization has proved tensegrity to be an optimal mass solution for various loading conditions (Skelton & Oliveira, 2009). Moreover, different prestress values correspond to different stiffness, allowing one to change the stiffness without changing the shape. These properties of tensegrity structure have led researchers to use tensegrity concepts in various applications from civil engineering bridges (Carpentieri, Skelton, & Fraternali, 2015) to soft-robotics (Karnan, Goyal, Majji, Skelton, & Singla, 2017; Sabelhaus, Akella, Ahmad, & SunSpiral, 2017) to various space applications; landers (Goyal et al., 2019; SunSpiral et al., 2013), deployable structures (Tibert & Pellegrino, 2003), and, space habitat designs (Goyal, Bryant, Majji, Skelton, & Longman, 2017).

Software Description

Modeling of Tensegrity Structures (MOTES) provides two categories for the analysis of any tensegrity structure. Firstly, static analysis provides the minimum mass of the tensegrity structure by optimizing tensile forces in the strings and compressive forces in the bars in the absence of external forces (self-equilibrium state), and in the presence of given external forces. In order to solve for the minimum mass required under yielding constraints, MOTES formulates the optimization problem as a “Linear Programming” problem. The software also allows solving for the minimum mass under buckling and yielding failure criteria through a non-linear optimization solver. Secondly, the dynamic analysis uses a second-order matrix differential equation to simulate the dynamics of any complexity of the tensegrity structure (Goyal & Skelton, 2019). This dynamic model assumes the bars to be rigid and strings to exhibit linear elastic behavior.
The software runs a modified Runge-Kutta integration package to solve the nonlinear differential equations. A bar length correction scheme is used to correct the dynamic response that might incur random errors because of computational limitations. This analytical correction step also restricts the errors in connection constraints for class-\(k\) structures. The class-\(k\), bar-to-bar connections (ball joints) are formulated as linear constraints. These constraints in the motion space give rise to constraint forces in the structure. An analytical solution is provided to solve for the constraint forces, and a reduced-order model is developed to simulate the dynamics in the restricted motion space. The mass in the strings is also included in the dynamics by discretizing the string into several point masses along the length of the string. This complete mathematical model for the tensegrity dynamics is developed in our recent dynamics paper (Goyal & Skelton, 2019).

**Advantages and Applications**

Most approaches in the field of multi-body dynamics use a minimal coordinate representation. This eliminates redundant variables as the body coordinates are used based on connection, from the first body to the last body in a particular order. One disadvantage of this approach is the use of transcendental functions, which can cause significant computational errors. Another disadvantage is the limited configuration of the rigid body connections to a topological tree. The proposed formulation uses a non-minimal coordinate system to describe the dynamics of the system. The approach uses 6 degrees of freedom (DOF) to define a rigid symmetric cylindrical bar with no rotation about the longitudinal axis. For a “class-\(k\)” tensegrity structure, there would be no moment or torque along the bar axis as the bars are connected by frictionless ball joints.

Moreover, all the strings are also connected to the node centers of the bars, resulting in no torque (no rotation) along the axis of the bar. The use of the non-minimal system results in a second-order matrix differential equation to describe the dynamics of any tensegrity system. One disadvantage of such a method is the violation of the rigid body (constant bar length) constraint in the presence of computational errors. In order to satisfy the constant bar length constraint, we use an analytical correction step to eliminate the approximation errors at each integration step. STEDY, a similar tensegrity simulator, uses a different correction step and formulation to run the dynamics (Tadiparthi, Hsu, & Bhattacharya, 2019). The added advantage with MOTES is the ability to model massive strings and to design minimum mass structures using static analysis.

The software is being used to develop various tensegrity structures like tensegrity robotic arm, tensegrity antenna, tensegrity lander, and space habitat, where we integrate structure and control design to get the required performance. The software was also used as a part of the class curriculum for the course ‘AERO-489/689 Design Elective: Advanced Statics and Dynamics of Flexible Structures: Tensegrity Systems’ which was offered in spring 2019 as an undergraduate-graduate class in the department of aerospace engineering at Texas A&M University, College Station.

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