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Special relativity in R: the `lorentz` package

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Software

- Review
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Introduction: the Lorentz transformation in special relativity

In special relativity (Einstein, 1905), the Lorentz transformations (Lorentz, 1904) supersede their classical equivalent, the Galilean transformations. Lorentz transformations operate on four-vectors such as the four-velocity or four-potential and are usually operationalised as multiplication by a 4×4 matrix. A Lorentz transformation takes the components of an arbitrary four-vector as observed in one coordinate system and returns the components observed in another system which is moving at constant velocity with respect to the first.

The materials have been made publicly available at: https://github.com/RobinHankin/lorentz and licensed under the GPL-3. To install the package, type

```
install.packages("lorentz")
library("lorentz")
```

at the `R` (R Core Team, 2022) command line.

Statement of need

Einstein's theory of special relativity presents specific difficulties for its teaching and learning (Prado et al., 2020). One particularly problematic concept is that of four velocity, defined as the deriviative of an object's four-displacement with respect to its proper time (Resnick, 1968). Students are often left puzzled as to why an object with three degrees of freedom is described using an object with four components. Observing (correctly) that the familiar classical velocity addition law is incorrect for both three-velocities and four-velocities in relativistic mechanics, students may reasonably ask in what way four-velocities are preferable to three-velocities.

The `lorentz` package (Hankin, 2022) furnishes a consistent suite of computational functionality to give numerical illustration of many concepts of special relativity, including manipulation of three- and four- velocities. It is accessible to undergraduates, being written in the `R` programming language (R Core Team, 2022) which will be familiar to many physics students. The package allows the user to manipulate Lorentz boosts using familiar matrix multiplication, and in addition incorporates an efficient and consistent interface to deal with many aspects of undergraduate-level relativity including active and passive transformations, four-momentum of photons, and in particular the difficult and nonintuitive behaviour of units in which $c \neq 1$ [such as S. I.]. The classical limit of $c \longrightarrow \infty$ is discussed explicitly and computationally. The package allows the user to employ efficient and natural vectorised R idiom in the context of relativistic kinematics.

The package was originally written to facilitate investigation into the nonassociative and noncommutative structure of three-velocities (Ungar, 1997); but the pedagogical impact of this is to reinforce the primacy of four-vectors in relativity. One student, using the package, succinctly offered the observation that "three-velocities suck": surely a profound insight.

Previous related work

There are a few existing software tools for working with Lorentz transformations, mostly developed in an educational context. Early work would include that of Horwitz et al. (1994), who describe `relLab`, a system for building a range of *gendanken* experiments in an interactive graphical environment. The author asserts that it runs on "any Macintosh computer with one megabyte of RAM or more" but it is not clear whether the software is still available. More modern contributions would include the `OpenRelativity` toolkit (Sherin et al., 2016) which simulates the effects of special relativity in the `Unity` game engine. There are also many excellent github repos that provide functionality to create simple visual displays of Lorentz transformations of events (nick1627 (2019) is a good example), although these are generally restricted to a single spatial dimension.

Educational models (in the sense of Pössel (2018)) for Einstein's general theory of relativity (Einstein, 1907) tend to be physical (Pössel, 2018); but software examples would include the present author's software (Hankin, 2021) for visualizing black holes and gravitational radiation.

The `lorentz` package: summary of high-level functionality

The `lorentz` package provides `R`-centric functionality for Lorentz transformations. It deals with formal Lorentz boosts and converts between three-velocities and four-velocities. Computational support for the nonassociative and noncommutative structure of relativistic three-velocity addition is included. Some functionality for relativistic transformation of tensors of order 2 such as the stress-energy tensor is given, with examples. In the package, the speed of light is 1 by default, but is user-settable and the classical limit is recovered by setting $c = \infty$. Both passive and active transformations are implemented. An extensive heuristic vignette detailing package idiom is included; to view this, type "`vignette("lorentz")`" at the `R` command line. There does not seem to be a known relativistic generalization of the classical distributive law $r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$ (Ungar, 1997). Ungar states that "It is hoped that one day [such a generalization] will be discovered. If exists, it is expected to be the standard distributive law relaxed by Thomas gyration in some unexpected way". The package is used to execute a systematic sweep through possible distributive laws consistent with Ungar's suggestion, unfortunately without success.

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