# Helper for Bézier Curves, Triangles, and Higher Order Objects

**Danny Hermes**[1]

**1** University of California, Berkeley

## Summary

`bezier` is a Python helper for Bézier curves and surfaces (T. W. Sederberg 2016, Farin (2001)). Bézier curves (and surfaces) are parametric curves with polynomial components, but they are expressed in terms of the Bernstein basis rather than the traditional power basis. In addition to being more numerically stable (R. Farouki and Rajan 1987, R. Farouki (1991), R. T. Farouki and Goodman (1996)), this basis allows "intuitive" manipulation of geometric shapes by controlling a set of points rather than via algebraic techniques.

Bézier curves and surfaces have been widely used for decades (Rida T. Farouki 2012) in industrial design (e.g. shape), computer fonts and graphics, mathematics (e.g. isoparametric elements in finite elements), and many other fields.

This library provides support for

- 2D plotting
- 2D intersection (via both geometric (T. Sederberg 1989, T. Sederberg and Nishita (1990), Kim, Lee, and Shin (1998), T. W. Sederberg and Parry (1986)) and algebraic (Jónsson and Vavasis 2005, Manocha and Demmel (1992)) algorithms)
- Curve and surface subdivision (R. T. Farouki and Neff 1990)
- Degree-elevation and reduction
- Evaluation of points on curves / surfaces
- Determining parameters corresponding to a point on a on a curve or on a surface (i.e. the inverse of evaluation)
- Specialization / reparameterization
- Self-intersection / singularity check for 2D surfaces
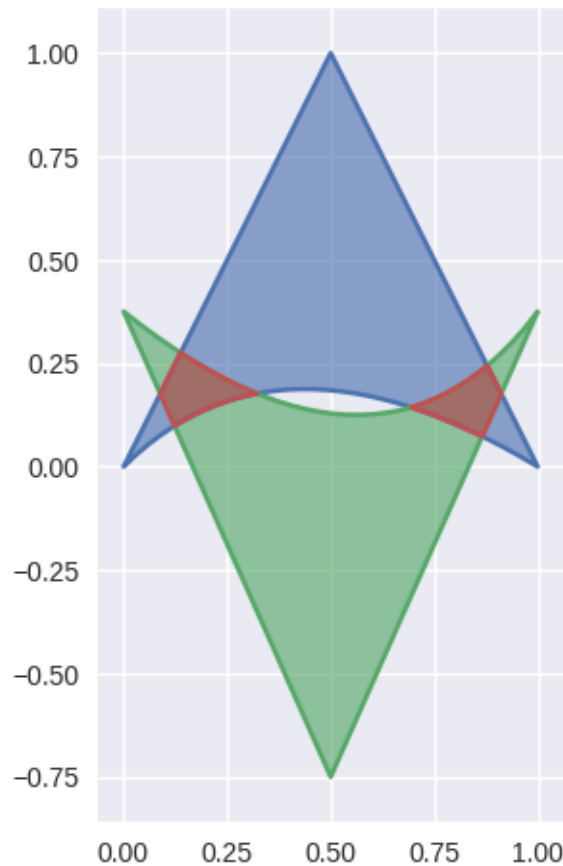
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