**ngsxfem: Add-on to NGSolve for geometrically unfitted finite element discretizations**

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**Summary**

ngsxfem is an add-on library to Netgen/NGSolve, a general purpose, high performance finite element library for the numerical solution of partial differential equations. The add-on enables the use of geometrically unfitted finite element technologies known under different labels, e.g. XFEM, CutFEM, TraceFEM, Finite Cell, fictitious domain method or Cut-Cell methods, etc. Both, Netgen/NGSolve and ngsxfem are written in C++ with a rich Python interface through which it is typically used. ngsxfem is an academic software. Its primary intention is to facilitate the development and validation of new numerical methods.

**Statement of need**

Typically, in the finite element method for the discretization of PDEs, the geometry under consideration is parametrized by the computational mesh yielding geometrically fitted finite element methods. The generation and adaptation of geometrically fitted computational meshes can be a burden on simulation methods, e.g. if the geometries are complex or especially if they are evolving in time. To be more flexible with regard to the geometry, geometrically unfitted finite element methods can be considered which break the direct link between the geometry parametrization and the computational mesh. Instead, a separate description of the geometry, e.g. through a level set function is used. ngsxfem aims at providing the necessary tools to robustly work in a geometrically unfitted setting where the geometry is described by one (or multiple) level set function(s). The essential tools extending standard finite element codes for the geometrically unfitted setting are:

- formulation of geometrically unfitted geometry through level set function(s)
- classification of elements in the computational mesh according to the unfitted geometry
- finite element spaces that consider the cut information
- tools to provide robust numerical integration on cut elements
- stabilization techniques to deal with arbitrary bad cuts (e.g. “ghost penalty”)

First of all ngsxfem provides these tools for Netgen/NGSolve. For other finite element frameworks similar libraries exist, e.g. libcutfem or multimesh for the FEniCS project, cf. Alnæs et al. (2015) and dune-udg, cf. Engwer & Heimann (2012), for dune, cf. Blatt et al. (2016) (more precisely dune-pdelab, cf. Bastian et al. (2010)). Let us also mention the FEMPAR finite element package which directly handles unfitted geometries, cf. Badia et al. (2018). In addition, ngsxfem has three advanced features beyond these, including:

- space-time finite elements and quadrature for unfitted space-time finite element discretizations of PDEs on moving domains, cf. Preuß (2018), Heimann (2020)

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the so-called direct version of the ghost penalty stabilization as introduced in Preuß (2018)

A more detailed overview of the key features provided by ngsxfem can be found in the repository under doc/feature-details.md. Furthermore, a detailed overview of the scientific literature which has utilised ngsxfem is given in the repository under doc/literature.md.

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References


