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adrt: approximate discrete Radon transform for Python

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Summary

The Radon transform is a fundamental integral transform that arises in many different fields including medical/seismic tomography, signal/image processing, and the analysis of partial differential equations (Natterer, 2001). The forward transform computes integrals over lines of an input image at various angles and offsets. This package implements a discretization of this transform called the approximate discrete Radon transform (ADRT), which computes integrals over pixel line segments allowing for a faster evaluation over digital images (Brady, 1998; Götz & Druckmüller, 1996). We provide an implementation of the ADRT and related transforms including a back-projection operation, a single-quadrant inverse, and the full multigrid inverse described in Press (2006). Each of these routines is accessible from Python, operates on NumPy arrays (Harris et al., 2020), and is implemented in C++ with optional OpenMP multithreading.

Statement of need

This package, `adrt`, aims to facilitate numerical experimentation with the ADRT by providing production-ready implementations of the ADRT algorithm and related transforms. We expect it to be useful in several broad respects: in scientific computing applications, in studying the properties of the ADRT, and in preparing new specialized software implementations.

The ADRT has demonstrated usefulness in scientific computing (Rim, 2018) and has applications in imaging, image processing, and machine learning that can benefit from the increased performance of the ADRT, which has a time complexity of $\mathcal{O}(N^2 \log N)$ for an $N \times N$ image (see Figure 1), compared to $\mathcal{O}(N^3)$ for the standard Radon transform (Press, 2006). The ADRT approximates the Radon transform with $\mathcal{O}(N^{-1} \log N)$ error, and it possesses important inversion properties that enable it to be used to approximate the inverse Radon transform (Press, 2006; Rim, 2020). Our documentation includes examples of the application of these routines to sample problems in tomography and PDEs, as well as recipes for implementing other transforms with our core routines, including an iterative inverse using the conjugate gradient iteration.

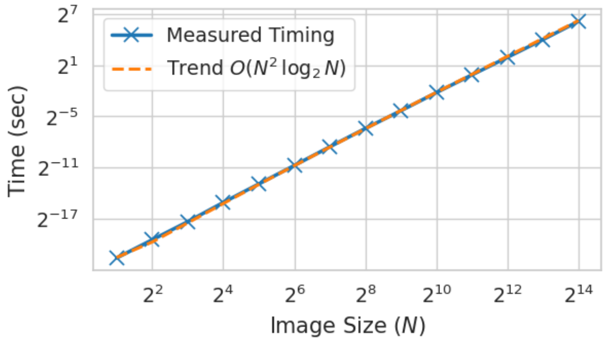

Figure 1: Running time of the ADRT for several image sizes with asymptotic trendline. Tests were run single-threaded on an Intel Xeon Platinum 8268 processor.

These routines also support research into the ADRT itself. While some private implementations exist (Bücker et al., 2015), to the best of our knowledge this is the only publicly available, open source implementation packaged for general use. This implementation provides a testbed for studying the ADRT, including routines exposing the progress of internal iterations. This package can also assist the development of specialized implementations, either by serving as a reference for new development or through reuse of the core C++ source, which is independent of Python.

Related research and software

A variety of other discretizations and approximations of the Radon transform exist, such as a linear interpolation and filtered back-projection in Walt et al. (2014), the discrete Radon transform (Beylkin, 1987), a fast transform based on the pseudo-polar Fourier transform (Averbuch et al., 2008), and the non-uniform fast Fourier transform (NUFFT) (Barnett et al., 2019; Greengard & Lee, 2004). However, the ADRT has unique properties that distinguish it from other discretizations, such as its localization property and range characterization (Li et al., 2023).

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