The Journal of Open Source Software

netroles: A Java library for role equivalence analysis in networks

Julian Müller [1,2]

1 Social Networks Lab, ETH Zürich, Switzerland **2** Institute of Computing, Università della Svizzera italiana, Switzerland

DOI: 10.21105/joss.05903

Software

- Review
- Repository
- Archive

Editor: Frederick Boehm

Reviewers:

- @leifeld
- @abhishektiwari

Submitted: 31 August 2023
Published: 17 January 2024

Summary

An important problem in the analysis of networks is structural similarity. It has commonly been expressed in terms of role equivalences, which have often been considered to formalize the concepts of social role and position as discussed by Linton (1936), Merton (1957) and Nadel (1957) in their analyses of social structure (Borgatti & Everett, 1992a). Role equivalences are based on the idea that actors are equivalent or play the same role if they form ties to similar others in similar ways. For example, the role of "doctor" is characterized by a set of ties to others playing related roles like "nurses", "patients" and "colleagues". However, this equivalence idea has been interpreted by different authors in different ways, resulting in the proposition of diverse definitions of role equivalence such as structural (Lorrain & White, 1971) or regular equivalence (White & Reitz, 1983). The `netroles` library provides implementations of many established notions of role equivalence, but more importantly, it offers a unified approach to role equivalence analysis that generalizes beyond the classic role equivalences, allowing users to express more complex kinds of role notions suitable for networks with multiple relations and attributes.

Statement of need

The general-purpose network analysis software `UCINET` (Borgatti et al., 2002), `visone` (Baur et al., 2002; Brandes & Wagner, 2004), and the R package `sna` (Butts, 2008) provide some methods to compute structural, regular, or regular equitable (Everett & Borgatti, 1996) equivalence as specific notions of role. More wide-spread are software tools that implement two kinds of relaxations of role equivalence:

- Distance or similarity measures based on structural, regular, or equitable equivalence are offered by software such as `blockmodeling` (Žiberna & Cugmas, 2023), `Pajek` (Batagelj & Mrvar, 2004), `sna` (Butts, 2008), and `UCINET` (Borgatti et al., 2002).
- *Blockmodeling* aims to find equivalences that best globally approximate a notion of role equivalence on a given network. Software like `graph-tool` (Peixoto, 2014), `sbm` (Chiquet et al., 2023), `blockmodels` (Leger et al., 2021), `StochBlock` (Škulj & Žiberna, 2022; Žiberna & Telarico, 2023), and `dynsbm` (Matias & Miele, 2020) offer *stochastic blockmodeling* methods derived from stochastic versions of structural equivalence. Packages such as `blockmodeling` (Žiberna, 2007; Žiberna & Cugmas, 2023), `dBlockmodeling` (Brusco et al., 2021; Brusco, 2023), `Pajek` (Batagelj & Mrvar, 2004), and `signnet` (Schoch, 2023) implement *optimizational blockmodeling* minimizing some criterion function based on one or more role equivalence concepts.

Nonetheless, only few tools offer partial support to compute a small selection of non-relaxed role equivalences, yet these solutions do not generalize to other kinds of role equivalences and are restricted to specific kinds of networks, such as unweighted networks or networks with categorical tie weights.

The `netroles` library is intended to close this gap and aims to extend the aim of applicability of role equivalences. It provides a unified framework for role equivalences based on the formalization of role equivalence proposed in Müller & Brandes (2022). The library implements established notions of role, but its main goal is to enable the expression of new role concepts, incorporating vertex and edge attributes as well as multiple relations according to the requirements of the network study. This also facilities the specification of error-tolerant role equivalences that allow for minor deviations from the ideal case. The example analyses in Müller & Brandes (2022) were conducted using `netroles`.

Background

The conceptual idea of role equivalence can be restated as a matching problem: two vertices i and j are role-equivalent if all neighbors or incident edges of i can be matched with neighbors or incident edges of j according to some set of matching rules and vice versa. These matching rules, in turn, frequently depend on some equivalence among the neighbors. Put differently, this defines an operation "relative role equivalence" that produces an equivalence by applying the matching rules relative to some given equivalence. Müller & Brandes (2022) showed that established role equivalences can be obtained as fixed points of relative role equivalence and the derived operations "role restriction" and "role extension", if suitable matching rules are chosen. The table below arranges established role equivalences supported by `netroles` by the underlying matching rules and the operations they are fixed points of.

Table 1: Established role equivalences as fixed points of the operations relative role equivalence, role restriction and role extension by type of matching rules.

matching type	matching rules for comparing i and j	relative role equiv.	role restriction	role extension
weak	no restrictions	weak equiv. (Winship & Mandel, 1983)	refinements of weak equiv.	coarsenings of weak equiv.
weakly equitable	each edge matches at most once	degree equality	refinements of degree equal.	coarsenings of degree equal.
strong structural	same neighbors	structural equiv. (Lorrain & White, 1971)	refinements of struct. equiv.	coarsenings of struct. equiv.
weak structural	tie (i, j) matched with tie (j, i) and (i, i) with (j, j), else same neighbors	weak structural equiv. (Everett et al., 1990)	refinements of weak struct. equiv.	coarsenings of weak struct. equiv.
regular	equivalent neighbors	perfect equiv. (Borgatti & Everett, 1994)	regular equiv. (White & Reitz, 1983)	ecological equiv. (Borgatti & Everett, 1992b)
equitable	equivalent neighbors, each edge matches at most once	equitable perfect equiv. (Everett & Borgatti, 1996)	equitable regular equiv. (Everett & Borgatti, 1996)	equitable ecological equiv. (Everett & Borgatti, 1996)

Design

The `netroles` library enables users to specify the relative role equivalence operation as appropriate. Inspired by dataflow and functional programming paradigms, users can define the

The Journal of Open Source Software

relative role equivalence operation through composition of simple operators in a declarative style. All other role operations described in Müller & Brandes (2022) are derived automatically. The focus on composition greatly increases the library's expressive power and improves its extensibility, as complex scenarios can be handled by breaking down the role definition into simpler suboperations.

To demonstrate this compositional design, we define an error-tolerant version of equitable equivalence, which considers two vertices role-equivalent if there is at most one deviation from the underlying matching rules for outgoing edges in each direction of comparison, as long as the compared vertices are not isolates. The resulting binary relation is transitively closed to regain an equivalence. The example in Section 5 of Müller & Brandes (2022) is based on this error-tolerant equivalence.

The dataflow diagram below describes this role operator graphically. Note that some intermediate results are not equivalences.

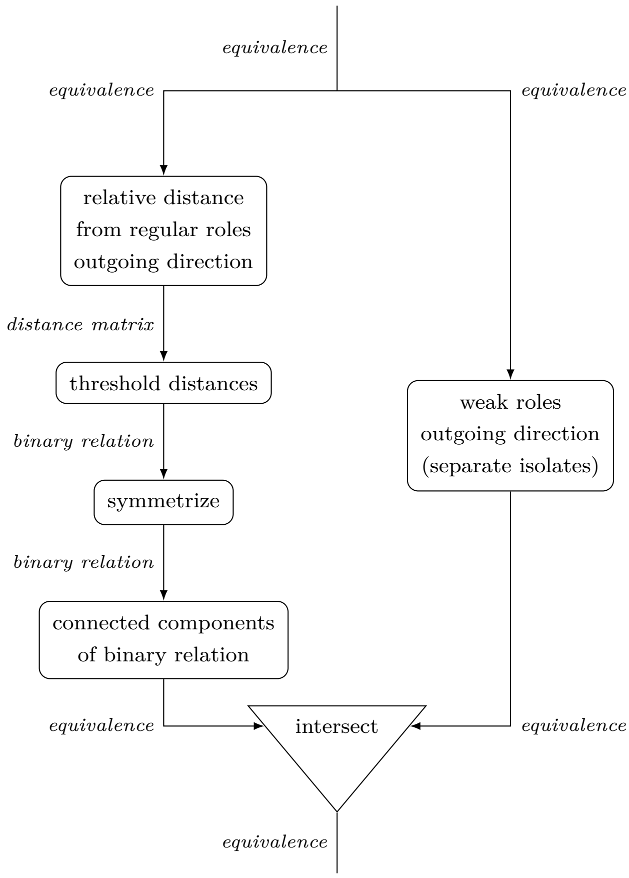

Figure 1: Dataflow diagram of error-tolerant role operator

The following code expresses this role operator using `netroles`:

```
RoleOperator<ConstMapping.OfInt> errortolerantOp = Operators.parallel(
  Reducers.EQUIVALENCE.meet(), // combine by intersecting equivalences
  Operators.composeRoleOp(
    Operators.composeOp(
      Operators.composeOp(
        // threshold pairwise deviations from equitable equivalence by one
        DistanceOperators.EQUIVALENCE.equitable()
```

```
          .of(NetworkView.fromNetworkRelation(network, Direction.OUTGOING))
          .make(),
        Converters.thresholdDistances((i, j) -> 1)),
      // symmetrize (at most one deviation in each directions)
      RoleOperators.BINARYRELATION.basic().symmetrize()),
    // close over pairs of role-equivalent vertices transitively
    Converters.strongComponentsAsEquivalence()),
  // and separate isolates
  RoleOperators.EQUIVALENCE.weak()
    .of(n, NetworkView.fromNetworkRelation(network, Direction.OUTGOING))
    .make());
```

References

Batagelj, V., & Mrvar, A. (2004). Pajek — analysis and visualization of large networks. In M. Jünger & P. Mutzel (Eds.), *Graph Drawing Software* (pp. 77–103). Springer. https://doi.org/10.1007/978-3-642-18638-7_4

Baur, M., Benkert, M., Brandes, U., Cornelsen, S., Gaertler, M., Köpf, B., Lerner, J., & Wagner, D. (2002). visone software for visual social network analysis. In P. Mutzel, M. Jünger, & S. Leipert (Eds.), *Graph Drawing. 9th International Symposium, GD 2001* (pp. 463–464). Springer. https://doi.org/10.1007/3-540-45848-4_47

Borgatti, S. P., & Everett, M. G. (1992a). Notions of position in social network analysis. *Sociological Methodology*, *22*, 1–35. https://doi.org/10.2307/270991

Borgatti, S. P., & Everett, M. G. (1992b). Graph colorings and power in experimental exchange networks. *Social Networks*, *14*(3-4), 287–308. https://doi.org/10.1016/0378-8733(92)90006-S

Borgatti, S. P., & Everett, M. G. (1994). Ecological and perfect colorings. *Social Networks*, *16*(1), 43–55. https://doi.org/10.1016/0378-8733(94)90010-8

Borgatti, S. P., Everett, M. G., & Freeman, L. C. (2002). *UCINET for Windows: Software for social network analysis*. Analytic Technologies.

Brandes, U., & Wagner, D. (2004). visone — analysis and visualization of social networks. In M. Jünger & P. Mutzel (Eds.), *Graph Drawing Software* (pp. 321–340). Springer. https://doi.org/10.1007/978-3-642-18638-7_15

Brusco, M. (2023). *dBlockmodeling: Deterministic blockmodeling of signed, one-mode and two-mode networks*. https://CRAN.R-project.org/package=dBlockmodeling

Brusco, M., Doreian, P., & Steinley, D. (2021). Deterministic blockmodelling of signed and two-mode networks: A tutorial with software and psychological examples. *British Journal of Mathematical and Statistical Psychology*, *74*(1), 34–63. https://doi.org/10.1111/bmsp.12192

Butts, C. T. (2008). Social network analysis with sna. *Journal of Statistical Software*, *24*(6). https://doi.org/10.18637/jss.v024.i06

Chiquet, J., Donnet, S., & Barbillon, P. (2023). *sbm: Stochastic blockmodels*. https://CRAN.R-project.org/package=sbm

Everett, M. G., & Borgatti, S. P. (1996). Exact colorations of graphs and digraphs. *Social Networks*, *18*(4), 319–331. https://doi.org/10.1016/0378-8733(95)00286-3

Everett, M. G., Boyd, J. P., & Borgatti, S. P. (1990). Ego-centered and local roles: A graph theoretic approach. *The Journal of Mathematical Sociology*, *15*(3-4), 163–172. https://doi.org/10.1080/0022250X.1990.9990067

Leger, J.-B., Barbillon, P., & Chiquet, J. (2021). *blockmodels: Latent and stochastic block model estimation by a 'V-EM' algorithm*. https://CRAN.R-project.org/package=blockmodels

Linton, R. (1936). *The study of man: An introduction*. Appleton-Century.

Lorrain, F., & White, H. C. (1971). Structural equivalence of individuals in social networks. *The Journal of Mathematical Sociology*, *1*(1), 49–80. https://doi.org/10.1080/0022250X.1971.9989788

Matias, C., & Miele, V. (2020). *dynsbm: Dynamic stochastic block models*. https://CRAN.R-project.org/package=dynsbm

Merton, R. K. (1957). *Social theory and social structure*. Free Press.

Müller, J., & Brandes, U. (2022). The evolution of roles. *Social Networks*, *68*, 195–208. https://doi.org/10.1016/j.socnet.2021.02.001

Nadel, S. F. (1957). *The theory of social structure*. Free Press.

Peixoto, T. P. (2014). The graph-tool python library. *Figshare*. https://doi.org/10.6084/m9.figshare.1164194

Schoch, D. (2023). signnet: An R package for analyzing signed networks. *Journal of Open Source Software*, *8*(81), 4987. https://doi.org/10.21105/joss.04987

Škulj, D., & Žiberna, A. (2022). Stochastic blockmodeling of linked networks. *Social Networks*, *70*, 240–252. https://doi.org/10.1016/j.socnet.2022.02.001

White, D. R., & Reitz, K. P. (1983). Graph and semigroup homomorphisms on networks of relations. *Social Networks*, *5*(2), 193–234. https://doi.org/10.1016/0378-8733(83)90025-4

Winship, C., & Mandel, M. (1983). Roles and positions: A critique and extension of the blockmodeling approach. *Sociological Methodology*, *14*, 314–344. https://doi.org/10.2307/270911

Žiberna, A. (2007). Generalized blockmodeling of valued networks. *Social Networks*, *29*(1), 105–126. https://doi.org/10.1016/j.socnet.2006.04.002

Žiberna, A., & Cugmas, M. (2023). *Generalized and classical blockmodeling of valued networks*. https://CRAN.R-project.org/package=blockmodeling

Žiberna, A., & Telarico, F. A. (2023). *Stochastic blockmodeling of one-mode and linked networks*. https://CRAN.R-project.org/package=StochBlock