

StochasticDominance.jl: A Julia Package for Higher Order Stochastic Dominance

Rajmadan Lakshmanan ¹ and Alois Pichler ¹

1 Faculty of Mathematics, University of Technology, Chemnitz, Germany

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Summary

Stochastic dominance plays a key role in decision-making under uncertainty and quantitative finance. It helps evaluate whether one investment, policy, or strategy is better than others in uncertain conditions. It provides a mathematically rigorous method often used in optimization to maximize returns or minimize risk. See Ogryczak & Ruszczyński (2002), Dentcheva & Ruszczyński (2003), Kuosmanen (2004), Post & Kopa (2017), Kopa et al. (2023), Maggioni & Pflug (2016), Maggioni & Pflug (2019), Consigli et al. (2023), and Pichler (2024) for an extensive body of research on the applicability of stochastic dominance.

Statement of need

Despite being a crucial tool, (higher-order) stochastic dominance involves infinitely many constraints, making it *computationally intractable* in practice. Our recent research Lakshmanan et al. (2025) addresses this challenge by theoretically reducing the infinite constraints to a finite number. However, no concrete, user-friendly implementation of (higher-order) stochastic dominance has been developed. Additionally, the existing prominent theoretical algorithms only discuss stochastic orders *two* and *three*, but not higher orders. Moreover, both, the discussion and implementation of non-integer orders, are absent.

To address this gap, we present *StochasticDominance.jl*, an open-source Julia package tailored for verification and optimization under higher-order stochastic dominance constraints.

Main features of the package

Technical highlights. The StochasticDominance.jl package offers robust functions for verifying higher-order stochastic dominance constraints between two random variables. It supports two primary objective functions by maximizing expected returns and minimizing higher-order risk measures to achieve the optimal asset allocation while satisfying higher-order stochastic dominance constraints. The package's optimization framework is built around Newton's method, which efficiently handles the non-linear constraints. To enhance efficiency, we first employ Particle Swarm Optimization (PSO), which approximates the solution over a set number of iterations. In our previous work, Lakshmanan et al. (2025) initially impose two fixed higher-order stochastic dominance constraints and dynamically introduce additional constraints to ensure dominance. To simplify the process and align with a black-box approach, this package uses these constraints with additional theoretical backing, eliminating the need for dynamic adjustments. Below, we provide a concise overview of its key functions.

1. verify_dominance: This function checks whether the given benchmark asset, represented as the random variable X, and the weighted portfolio asset, represented as the random variable Y, exhibit a dominance relationship for the specified stochastic order.



- 2. optimize_max_return_SD: This function determines the optimal asset allocation that maximizes expected returns for a given stochastic order (SDorder).
- optimize_min_riskreturn_SD: This function determines the optimal asset allocation by minimizing higher-order risk measures for a given stochastic order (SDorder) while also indicating whether dominance is achieved. Further implementation details and various examples are available in the package's documentation.

Example: verification and portfolio optimization

The first example demonstrates how to use the verify_dominance function to check stochastic dominance of a specified order.

julia> using StochasticDominance# Load the packagejulia> Y = [3, 5, 7, 9, 11]# Random variable Yjulia> p_Y = [0.15, 0.25, 0.30, 0.20, 0.10]# Probabilities associated with Yjulia> X = [2, 4, 6, 8, 10]# Random variable Xjulia> p_X = [0.10, 0.30, 0.30, 0.20, 0.10]# Probabilities associated with X

Next, define the stochastic order and execute the function as shown below

```
julia> SDorder = 2
julia> verify_dominance(Y, X, SDorder; p_Y, p_X)
Y dominates X in stochastic order 2
true
```

This function checks whether Y stochastically dominates X of order SDorder.

Optimization: maximize expected return

Next, we demonstrate how to find the optimal allocation that maximizes the expected return of the portfolios of interest while satisfying stochastic dominance of a given order. See tutorials for a comprehensive technical explanation.

The data.csv file is located in the /test folder of the code repository.

Use the following function to compute the optimal allocation (objective: maximize expected return):

From x_opt, we obtain the optimal asset allocation. Enabling plot=true generates graphical representations summarizing key insights concisely.





Figure 1: Optimal asset allocation (SD order 4.0). The chart illustrates the proportion of different assets in an optimized portfolio. The maximized expected return is 0.304%, compared to a benchmark return of 0.125%.

If no portfolio allocation satisfies the stochastic dominance constraint of the given order, the algorithm stops and provides the necessary information.

Optimization: minimizing higher-order risk measures

Next, we demonstrate how to determine the optimal allocation that minimizes higher-order risk measures while ensuring stochastic dominance of a given order for the portfolios of interest. See tutorials for a detailed technical explanation.

Use the following function to compute the optimal allocation (objective: minimizing higher-order risk measure):

```
julia> SDorder = 4.7;

julia> x_opt, q_opt, t_opt = optimize_min_riskreturn_SD(

\xi,

\xi_0,

SDorder;

p_\xi = fill(1 / n, n), # Uniform probability

p_\xi_0 = fill(1 / n, n), # Uniform probability

\beta = 0.5, # Risk parameter

r = 2.0, # Order of the risk measure

plot=true,

)
```

From q_opt, we derive the value of optimal parameter that satisfies the risk measure for the given portfolio. It is important to note that the objective itself is an optimization problem. However, our algorithm is designed to compute both simultaneously in a single execution. Additionally, the algorithm supports non-integer stochastic dominance orders. This feature is not restricted to this single setup but is available across all functions.



Figure 2: Optimal asset allocation (SD order 4.7). The pie chart illustrates the distribution of assets in the optimized portfolio. The optimal coherent risk-return balance results in a portfolio return of 0.97%, compared to a benchmark return of 1.167%.



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