GREOPy: A Python package for solving the emitter-observer problem in general relativity

Jan P. Hackstein [1¶] **and Eva Hackmann** [1]

1 Center of Applied Space Technology and Microgravity (ZARM), University of Bremen, Am Fallturm 2, 28359 Bremen, Germany ¶ Corresponding author

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Summary

Many modern methods of observation and long-range communication rely on the exchange of electromagnetic signals to transmit information. To simulate such a signal exchange, the emission and the corresponding observation points in space and time have to be calculated via the signal sent between them, which is called the Emitter-Observer Problem (EOP). A prominent example in astrophysics is the stationary EOP, where all light rays connecting a distant light source to an observer, neither moving in space, are calculated taking into account effects on the light propagation due to gravity, (see e.g. Viergutz, 1993). In this case, the EOP is characterized by the boundary-value problem (BVP) between the emission event at some light source and the observer. However, the EOP is a much more general problem also present in satellite communication, e.g. for quantum key distribution (Forges de Parny et al., 2023; Khatri et al., 2021; Sidhu et al., 2021) and relativistic geodesy (Müller et al., 2018; Sheard et al., 2012; Tapley et al., 2004). Relative motion between communicating satellites makes the EOP a non-stationary problem requiring a more nuanced approach to the BVP. If a massive object is present in the setup, signals passing between emitter and observer are additionally influenced by the underlying gravitational field of this object, undergoing e.g. gravitational redshift and light bending. The most accurate theory to date to describe such effects is the theory of general relativity (GR). The numerical implementation of the EOP presented here provides a fully relativistic framework that considers both kinematical and gravitational effects of each setup in question.

Statement of need

The *General-Relativistic Emitter-Observer problem Python algorithm* (`GREOPy`) is a Python package for solving the EOP in the GR framework, meaning in either the absence or presence of massive objects. More precisely, it is an implementation of the numerical shooting method (Press, 2007) that solves the BVP posed by communication between two (potentially) moving objects such as satellites or ground stations by converting it into a set of initial-value problems (IVPs) with varying initial conditions. The IVP corresponds to solving the geodesic equation in GR, i.e. the equation of motion describing the light signal propagation through spacetime. To that end, `GREOPy` employs the NumPy (Harris et al., 2020), SciPy (Virtanen et al., 2020) and SymPy (Meurer et al., 2017) libraries for the implementation of the spacetime metric, the geodesic equation, and use of solvers such as `solve_ivp`. Both communicating objects are assumed to be test particles that generate no gravitational field of their own. The underlying geometry of spacetime is in principle arbitrary and may be chosen freely. The Schwarzschild and the post-Newtonian spacetime to first order have already been implemented in the code base, enabling the study of the EOP for many inter-satellite as well as ground station-satellite configurations. However, `GREOPy` was written in a modular manner to allow for easy implementation of additional spacetimes by the community, and further additions are planned to be implemented.

The calculation of celestial orbits is already present within the Python ecosystem. One notable library is Gala (Price-Whelan, 2017) for galactic dynamics. Particle trajectories in fields of moons, planets or the sun are covered by e.g. the ASSIST package (Holman et al., 2023) for ephemeris-quality integration, implemented in C99 with a Python wrapper for its underlying functions. Additionally, the Black Hole Perturbation Toolkit (*Black Hole Perturbation Toolkit*, 2025) and their related projects cover a broad range of problems in GR and gravitational physics in- and outside of the Python ecosystem, focusing mostly on the Kerr spacetime, which is most relevant for black holes. Similarly, the EinsteinPy (Bapat et al., 2021) package provides functionalities to calculate geodesics for both massive particles and light, but again focusing on the Schwarzschild and Kerr(-Newman) spacetimes. An implementation in the Wolfram language of analytically given solutions for geodesics of massive particles and light in Kerr spacetimes is e.g. given in (Cieślik et al., 2023). Another related software tool is squirrel.jl (Feng et al., 2022) written in the Julia language, which implements the localization of events in curved spacetime for use in relativistic positioning systems. It calculates the intersection of the light cones of events on four different satellite trajectories.

Owing to the already available packages regarding particle trajectory calculations in GR, `GREOPy` does not focus on orbit computation, but allows for importing orbits from outside the package, and offers a small tool for creating simplified geodesic test orbits. `GREOPy` stands out from the above-mentioned packages by specifically solving the fully relativistic EOP in an operationally meaningful manner using direct pairwise satellite communication, a feature for which no Python package seems to exist.

In GR, clocks on different heights in a gravity potential will show a mutual gravitational redshift or, equivalently, gravitational time dilation. Recent technological advancements enable the use of highly-stable optical clocks on Earth (Mehlstäubler et al., 2018) to determine from these redshifts height differences in the centimeter regime. In space, the same effect will lead to a redshift between clocks on different satellites or on ground stations. `GREOPy` was developed for the purpose of simulating general-relativistic redshifts between clocks on satellites, with the aim to extract height information. This is done by connecting the clocks on the satellites by a light signal to evaluate the redshift. Towards this end, the EOP is solved in the full non-linear general-relativistic framework, without resorting to a post-Newtonian approximation of the signal path. The `GREOPy` package is therefore specifically aimed at researchers simulating light signals around massive objects, such as the Earth, to utilize the full extent of the general-relativistic framework.

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