




# COSMOS: A numerical relativity code specialized for PBH formation

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## Summary

Primordial black holes (PBHs) are black holes generated in the early universe without having gone through stellar evolution. It has been hypothesized that PBHs may be candidates for black holes and compact objects of various masses in the universe or a major component of dark matter. In particular, PBHs have been attracting much attention in the recent development of gravitational wave observation. In the standard formation process, PBHs are formed from super-horizon primordial fluctuations with non-linearly large initial amplitude. In order to simulate the non-linear gravitational dynamics of PBH formation, one has to rely on numerical relativity solvers to approximate the solution of the Einstein equations. [COSMOS](#) ([Okawa et al., 2014](#); [Yoo et al., 2013](#)) provides simple tools for the simulation of PBH formation (see [COSMOS-S](#) for a spherically symmetric version of COSMOS, which is not discussed in this paper). COSMOS is a C++ package for solving the Einstein equations in 3+1 dimensions. It was originally translated from SACRA code ([Yamamoto et al., 2008](#)) into C++ and has been developed specifically for the simulation of PBH formation.<sup>1</sup> Past publications that use COSMOS for simulation include [Yoo et al. \(2013\)](#), [Okawa et al. \(2014\)](#), [Yoo & Okawa \(2014\)](#), [Okawa & Cardoso \(2014\)](#), [Ikeda et al. \(2015\)](#), [Brito et al. \(2015\)](#), [Brito et al. \(2016\)](#), [Okawa \(2015\)](#), [Yoo et al. \(2017\)](#), [Yoo et al. \(2019\)](#), [Yoo \(2024\)](#), [Escrivà & Yoo \(2025b\)](#) and [Escrivà & Yoo \(2025a\)](#).<sup>2</sup>

## Statement of need

In the simulation of PBH formation, the presence of multiple length scales (the size of the collapsing region and that of cosmological expansion) necessitates an efficient resolution refinement procedure. In order to resolve the collapsing region, non-Cartesian scale-up coordinates ([Yoo et al., 2019](#)) and a fixed mesh-refinement procedure ([Yoo, 2024](#)) are implemented in COSMOS. In its model, COSMOS uses a perfect fluid with a linear equation of state and a massless scalar field as matter fields. To achieve a practically acceptable computational speed, OpenMP is used for the parallelization. COSMOS has no other dependencies, which makes for an easier installation. Once users understand the source

<sup>1</sup>C.Y. and H.O. are the main contributors of this code, and other authors used the numerical code during the development and operational stages and contributed in part to its development and improvement.

<sup>2</sup>In these works, additional functions and packages have been implemented that may not appear in the public release of COSMOS. Therefore, the results may not be obtained by simply running the public code.

code to some extent, the system can be easily extended to various scientifically interesting settings.

## Physical system settings

At the core of COSMOS, the Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

are solved, where  $G_{\mu\nu}$ ,  $g_{\mu\nu}$ ,  $R_{\mu\nu}$ ,  $R$ ,  $G$ ,  $c$  and  $T_{\mu\nu}$  are the Einstein tensor, the metric tensor, the Ricci tensor, the Ricci scalar, the Newtonian gravitational constant, the speed of light, and the energy-momentum tensor, respectively. The energy-momentum tensor is divided into fluid and scalar field contributions as

$$T_{\mu\nu} = T_{\mu\nu}^{\text{SC}} + T_{\mu\nu}^{\text{FL}},$$

with

$$T_{\mu\nu}^{\text{SC}} = \nabla_{\mu}\phi\nabla_{\nu}\phi - \frac{1}{2}g_{\mu\nu}\nabla^{\lambda}\phi\nabla_{\lambda}\phi$$

and

$$T_{\mu\nu}^{\text{FL}} = (\rho + P)u_{\mu}u_{\nu} + Pg_{\mu\nu},$$

where  $\nabla$  denotes taking the covariant derivative using  $g_{\mu\nu}$ ,  $\phi$  is the scalar field, and  $\rho$ ,  $u_{\mu}$  and  $P$  are the energy density, the four-velocity, and the pressure of the fluid, respectively. The pressure and the energy density are assumed to satisfy the linear equation of state  $P = w\rho$  with  $w$  being a constant. The equations of motion for the scalar field

$$\nabla^{\mu}\nabla_{\mu}\phi = 0,$$

and the fluid

$$\nabla^{\mu}T_{\mu\nu}^{\text{FL}} = 0,$$

are also solved. Readers are asked to refer to standard textbooks of numerical relativity (e.g., [Gourgoulhon, 2012](#); [Shibata, 2016](#)) on how to rewrite these equations into a form suitable for numerical integration. To solve the fluid equations of motion, we basically follow the scheme discussed in [Kurganov & Tadmor \(2000\)](#) and [Shibata & Font \(2005\)](#).

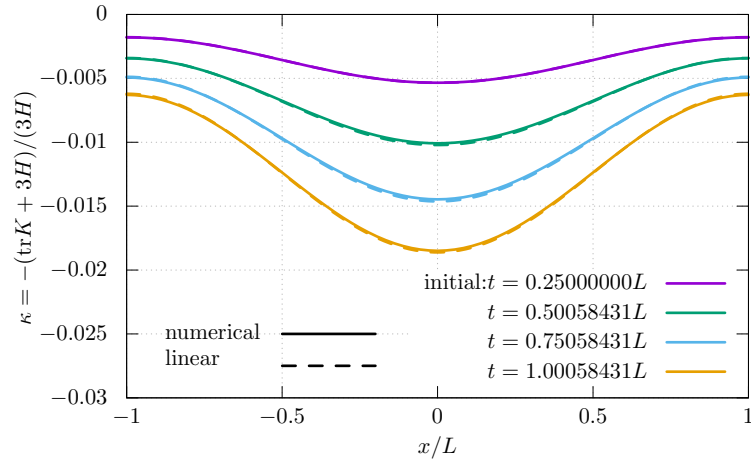
For the initial data, we adopt the long-wavelength growing-mode solutions up to (and including) the next-leading order of the expansion parameter  $\epsilon = k/(aH) \ll 1$ , where  $1/k$  gives the characteristic comoving scale of the inhomogeneity, and  $a$  and  $H$  are the scale factor and Hubble expansion rate in the reference background universe. The initial data can be characterized by a function of the spatial coordinates  $\vec{x}$  as the curvature perturbation  $\zeta(\vec{x})$  for adiabatic fluctuations ([Harada et al., 2015](#); [Yoo et al., 2020](#); [Yoo, 2024](#)) and iso-curvature perturbation  $\Upsilon(\vec{x})$  for massless scalar iso-curvature ([Yoo et al., 2022](#)). Since the space is filled with the fluid, the initial fluid distribution can be generated to meet the constraint equations included in the Einstein equations. Therefore, the constraint equations are initially satisfied to within machine precision, and need not be solved by integrating elliptic differential equations. This approach differs from the standard method of obtaining the initial data for spacetimes with asymptotically flat vacuum regions, and is the reason why elliptic solvers are not included in COSMOS.

## Examples

Three examples are included in the package. These examples are intended primarily for demonstration and instructional purposes, and thus they are not highly accurate. The resolution has been intentionally kept as low as possible. In the figures below, the length scale is normalized by the size  $L$  of the box for the numerical simulation.

### Evolution of a single-mode perturbation

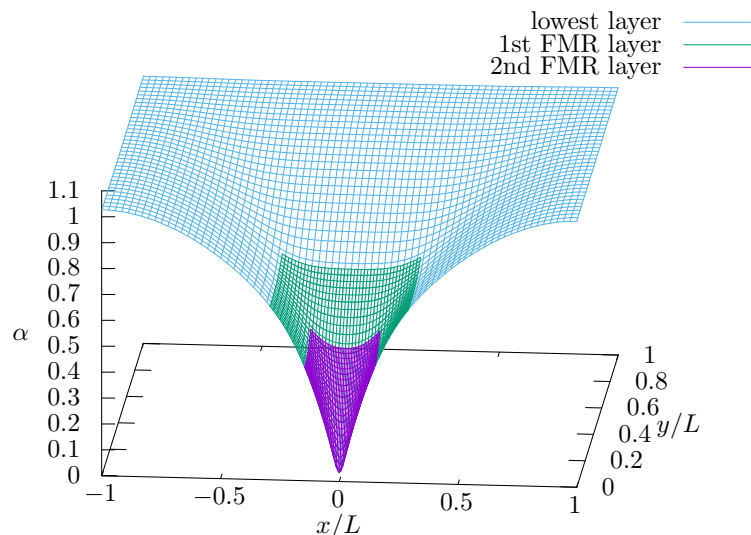
The evolution of a small sinusoidal fluctuation is given as an example, which can be compared to the corresponding linear perturbation (see [Figure 1](#)).



**Figure 1:** The time evolution of the trace of the extrinsic curvature  $\text{tr}K$  is compared with the solution of the linear perturbation equation.

### Adiabatic spherically symmetric initial fluctuation

Here, we consider the adiabatic perturbation generated by the initial curvature perturbation without scalar field contribution. As is described in Harada et al. (2015), once the spatial profile of the curvature perturbation  $\zeta(\vec{x})$  is specified, the growing mode solution can be described in the long-wavelength approximation. More details including the specific functional form of the curvature perturbation can be found in the [instruction page](#) (see also Yoo et al., 2020). In the repository, we include the data file `ini_all.dat` necessary to reconstruct the geometry and matter distribution at the time an apparent horizon is found (see [Figure 2](#) and [Figure 3](#) which can be generated by following the instructions).



**Figure 2:** The lapse function (“ $tt$ -component” of the metric) on the  $xy$ -plane at the time when an apparent horizon is found. The blue, green, and purple meshes show the region covered by the lowest, first, and second mesh refinement layers, respectively.

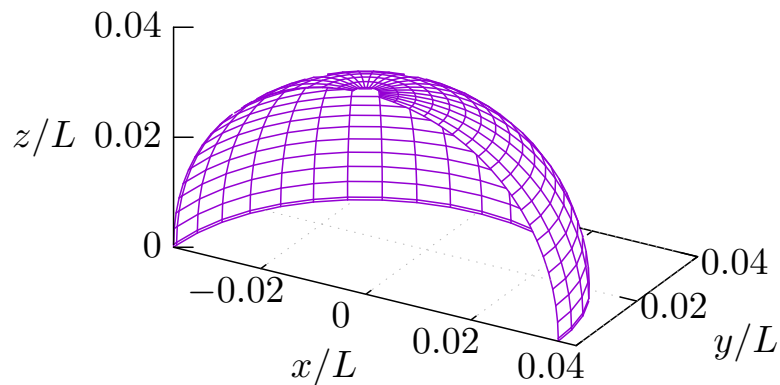


Figure 3: The shape of the apparent horizon when it is found.

### Spherically symmetric iso-curvature

Here, we consider the iso-curvature perturbation generated by a massless scalar field. We assume that the massless scalar field does not contribute to the background metric of the gradient expansion. Then, as is described in Yoo et al. (2022), once the spatial profile of the iso-curvature mode  $\Upsilon(\vec{x})$  is specified at the leading order of the gradient expansion, the growing mode solution can be described in the long-wavelength approximation. More details including the specific functional form of  $\Upsilon(\vec{x})$  can be found in the [instruction page](#) (see also Yoo et al., 2022). In the repository, we include the data file `ini_all.dat` necessary to reconstruct the geometry and matter distribution at the time an apparent horizon is found, as for the adiabatic case.

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### References

- Brito, R., Cardoso, V., Macedo, C. F. B., Okawa, H., & Palenzuela, C. (2016). Interaction between bosonic dark matter and stars. *Physical Review D*, 93(4), 044045. <https://doi.org/10.1103/PhysRevD.93.044045>
- Brito, R., Cardoso, V., & Okawa, H. (2015). Accretion of dark matter by stars. *Physical Review Letters*, 115(11), 111301. <https://doi.org/10.1103/PhysRevLett.115.111301>
- Escrivà, A., & Yoo, C.-M. (2025a). Nonspherical effects on the mass function of primordial black holes. *Physical Review D*, 112(8), L081304. <https://doi.org/10.1103/4jbp-87wc>
- Escrivà, A., & Yoo, C.-M. (2025b). Simulations of ellipsoidal primordial black hole formation. *Physical Review D*, 112(8), 083518. <https://doi.org/10.1103/PhysRevD.112.083518>
- Gourgoulhon, E. (2012). *3+1 formalism in general relativity : Bases of numerical relativity*. Springer. <https://doi.org/10.1007/978-3-642-24525-1>

- Harada, T., Yoo, C.-M., Nakama, T., & Koga, Y. (2015). Cosmological long-wavelength solutions and primordial black hole formation. *Physical Review D*, *D91*(8), 084057. <https://doi.org/10.1103/PhysRevD.91.084057>
- Ikeda, T., Yoo, C.-M., & Nambu, Y. (2015). Expanding universe with nonlinear gravitational waves. *Physical Review D*, *92*(4), 044041. <https://doi.org/10.1103/PhysRevD.92.044041>
- Kurganov, A., & Tadmor, E. (2000). New high-resolution central schemes for nonlinear conservation laws and convection–diffusion equations. *Journal of Computational Physics*, *160*, 241–282. <https://doi.org/10.1006/jcph.2000.6459>
- Okawa, H. (2015). Nonlinear evolutions of bosonic clouds around black holes. *Classical and Quantum Gravity*, *32*(21), 214003. <https://doi.org/10.1088/0264-9381/32/21/214003>
- Okawa, H., & Cardoso, V. (2014). Black holes and fundamental fields: Hair, kicks, and a gravitational Magnus effect. *Physical Review D*, *90*(10), 104040. <https://doi.org/10.1103/PhysRevD.90.104040>
- Okawa, H., Witek, H., & Cardoso, V. (2014). Black holes and fundamental fields in numerical relativity: initial data construction and evolution of bound states. *Physical Review D*, *89*(10), 104032. <https://doi.org/10.1103/PhysRevD.89.104032>
- Shibata, M. (2016). *Numerical relativity*. World Scientific Publishing Co. Pte. Ltd. <https://doi.org/10.1142/9692>
- Shibata, M., & Font, J. A. (2005). Robustness of a high-resolution central scheme for hydrodynamic simulations in full general relativity. *Physical Review D*, *72*, 047501. <https://doi.org/10.1103/PhysRevD.72.047501>
- Yamamoto, T., Shibata, M., & Taniguchi, K. (2008). Simulating coalescing compact binaries by a new code SACRA. *Physical Review D*, *78*, 064054. <https://doi.org/10.1103/PhysRevD.78.064054>
- Yoo, C.-M. (2024). Primordial black hole formation from a nonspherical density profile with a misaligned deformation tensor. *Physical Review D*, *110*(4), 043526. <https://doi.org/10.1103/PhysRevD.110.043526>
- Yoo, C.-M., Harada, T., Hirano, S., Okawa, H., & Sasaki, M. (2022). Primordial black hole formation from massless scalar isocurvature. *Physical Review D*, *105*(10), 103538. <https://doi.org/10.1103/PhysRevD.105.103538>
- Yoo, C.-M., Harada, T., & Okawa, H. (2017). 3D simulation of spindle gravitational collapse of a collisionless particle system. *Classical and Quantum Gravity*, *34*(10), 105010. <https://doi.org/10.1088/1361-6382/aa6ad5>
- Yoo, C.-M., Harada, T., & Okawa, H. (2020). Threshold of primordial black hole formation in nonspherical collapse. *Physical Review D*, *102*(4), 043526. <https://doi.org/10.1103/PhysRevD.102.043526>
- Yoo, C.-M., Ikeda, T., & Okawa, H. (2019). Gravitational collapse of a massless scalar field in a periodic box. *Classical and Quantum Gravity*, *36*(7), 075004. <https://doi.org/10.1088/1361-6382/ab06e2>
- Yoo, C.-M., & Okawa, H. (2014). Black hole universe with a cosmological constant. *Physical Review D*, *89*(12), 123502. <https://doi.org/10.1103/PhysRevD.89.123502>
- Yoo, C.-M., Okawa, H., & Nakao, K. (2013). Black hole universe: Time evolution. *Physical Review Letters*, *111*, 161102. <https://doi.org/10.1103/PhysRevLett.111.161102>